Quiz 13

March 22, 2017

Show all work and circle your final answer.

1. Use the alternating series estimation theorem to find the smallest value of n for which the partial sum s_n approximates $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n^2} = \sum_{n=2}^{\infty} \frac{(-1)^n}{n^2}$ correct to 2 decimal places. $\longrightarrow$ error $< .005$

(converges by AST)

The alternating series estimation theorem says $|s - s_n| \leq b_{n+1}$

$b_{n+1} = \frac{1}{(n+1)^2} \overset{\text{set}}{<} .005$

$\frac{1}{.005} < (n+1)^2$

$\frac{1000}{5} < (n+1)^2$

$200 < (n+1)^2$

$15 \leq n+1 \quad (14^2 = 196,\ 15^2 = 225)$

$14 \leq n$

$\boxed{n = 14}$ is the smallest value of n

2. Are the following series convergent or divergent? State the test you used and check all conditions.

(a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ This is an alternating series, so use AST:

$b_n = \frac{1}{\ln n}$

(1) $\frac{1}{\ln(n+1)} < \frac{1}{\ln n}$, so b_n is decreasing ✓

(2) $\lim_{n \to \infty} \frac{1}{\ln n} = 0$ ✓

$\boxed{\text{converges by AST}}$

(b) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ (Hint: $\ln n < n$)

compare to $\sum_{n=2}^{\infty} \frac{1}{n}$:

$\frac{1}{\ln n} > 0$ for $n \geq 2$ and $\frac{1}{n} > 0$ for $n \geq 2$ ✓

$\frac{1}{\ln n} > \frac{1}{n}$ since $\ln n < n$ ✓

$\sum_{n=2}^{\infty} \frac{1}{n}$ diverges by p-series / harmonic series ✓

$\sum_{n=2}^{\infty} \frac{1}{\ln n}$ $\boxed{\text{diverges by DCT}}$ with $\sum_{n=2}^{\infty} \frac{1}{n}$.